PULSED ACTION OF INTENSE HEAT FLUXES AND VOLUME HEAT SOURCES ON METALS

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When a high-power thermal pulse acts on a metal, evaporation is one of the chief effects taking place during this thermophysical process. The temperature field of the metal is considered, allowing for evaporation, when both the surface and interior of the metal (or the surface alone) are exposed to intense thermal action. The results obtained from machine and analytic solution of the corresponding problems are given.

In practice, a high-power energy pulse can be directed at a metal through use of a plasma jet, a laser beam, or a pulse discharge. No matter what method is used, the introduction of energy into the metal gives rise to a thermophysical process with certain characteristic features. One such feature is associated with electron-gas functions while another involves breakdown of the condensed phase on the metal surface.

For both radiative [1, 2] and electric action [3–5], the energy received by the metal is primarily absorbed by a gas of valence electrons. This energy is then transferred to the ion lattice by means of a relaxation mechanism. The resulting relative difference in the temperature Θ of the electron gas and the temperature T of the ion lattice

$$\varepsilon = \frac{\Theta - T}{T} \tag{1}$$

takes on values depending on the pulse power; however, when energy ceases to be supplied, $\varepsilon \to 0$ as $t \to \tau$, where τ is the relaxation time. As given in [4, 2], this last quantity has values on the order of 10^{-11} sec.

Since the duration of pulses feasible in practice is many orders greater than the relaxation time, ε assumes its steady-state value $\varepsilon = \varepsilon_0$ at the very beginning of the pulse. As noted in [2], during radiative action $\varepsilon_0 \sim F$, and for $F = 10^{12}$ W/m^2, $\varepsilon_0 \ll 1$. During electrical action in accordance with the equation $\varepsilon_0 \sim j^2$ obtained in [4] and for $j = 5 \cdot 10^7$ A/cm^2, ε_0 varies for different metals within the limits of 10^{-3} to 10^{-2}. For such values of ε_0 it is quite permissible to calculate the thermophysical process in a metal according to its ion temperature T.

The relaxation mechanism ensures effective transfer of energy from the electron gas to the ion lattice, but it is local in nature. Locally released energy is shifted beyond the corresponding region by a transfer mechanism in which both electron and phonon gases participate. However, the phonon component of heat conduction in the temperature region considered ($\sim 10^4$ °K) is more than two orders smaller than the electron component and, consequently, the total heat conduction of the metal under such conditions can be represented by its electron component alone.

A metal subjected to the action of a high-power thermal pulse is heated to a high temperature and very intense evaporation develops on its surface. In [2, 6] the Frenkel evaporation mechanism [8, 9] was used to describe this process. According to this mechanism the specific evaporation power and front velocity are expressed by the equations

$$F_{\text{evap}} = r_V v = F_0 \exp(-T_m/T), \quad v = v_0 \exp(-T_m/T), \tag{2}$$

where r_V and T are, respectively, the specific volume heat of vaporization and the temperature of the front.

According to (2), the thermophysical problem during pulsed action on a metal, with allowance for energy losses by evaporation, must be formulated in terms of a moving front with an appropriately chosen coordinate origin at the evaporation front itself.

First consider a pulsed thermophysical process excited by the action of a laser beam. As noted in [2], the unreflected part of the radiant flux is almost completely absorbed by the surface layer of the metal, whose thickness is only 10^{-5} to 10^{-6} cm. For such an essentially surface thermal effect the unreflected radiant flux F can be represented as a boundary condition of the second kind, specified at the moving front and with allowance for its losses by evaporation. Therefore, the corresponding one-dimensional problem when the coordinate origin is taken at the evaporation front can be formulated as:

$$\frac{\partial T(x, t)}{\partial t} = a \frac{\partial^2 T(x, t)}{\partial x^2} + v_0 \exp\left[-\frac{T_m}{T(0, t)}\right] \frac{\partial T(x, t)}{\partial x}, \tag{3}$$

Table 1

Steady-state Values of the Evaporation-Front Velocity and Temperature for Cd with Different Thermal-Flux Densities

F, W/m^2	10^{10}	10^{11}	10^{12}	10^{13}
v^*, m/sec	0,94	8.2	66.5	445
T^*, °K	1650	2250	3750	7350

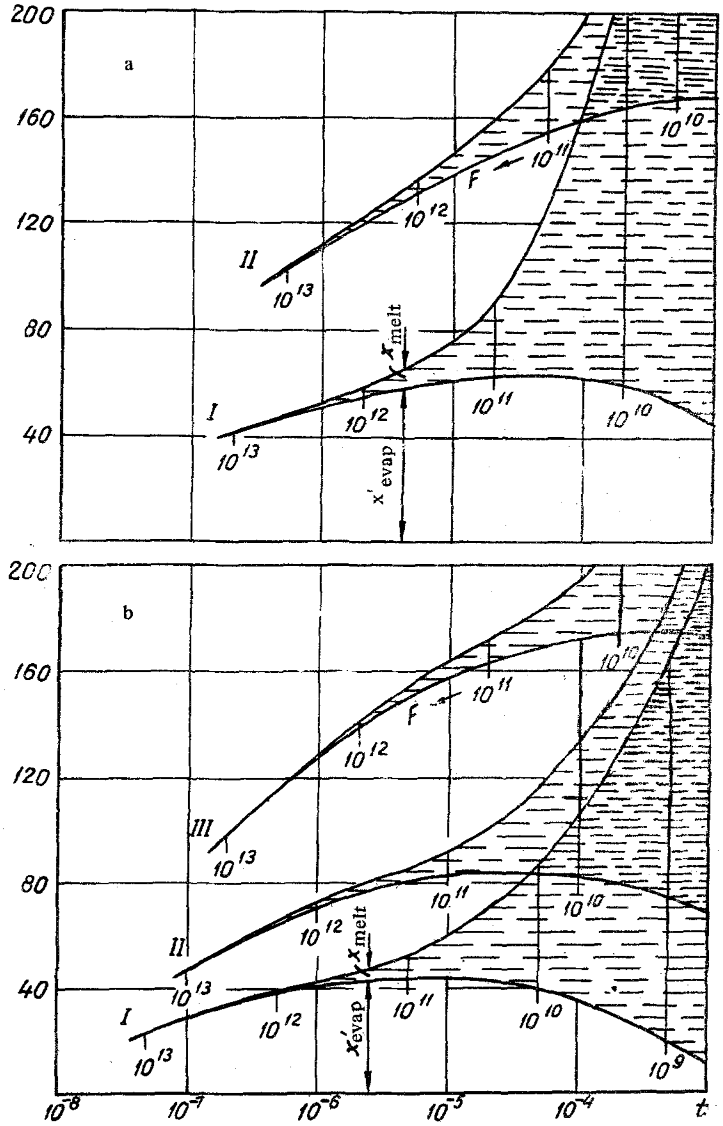

Fig. 1. Effect of the time (t, sec) a pulse of the energy W_S acts on a metal on the final position of the evaporation front and the melting isotherm (x'_{evap}, $x'_{evap} + x_{melt}$, μ; F, W/mm^2); a) Sn; W_S = 2 (I) and 5 (II), J/mm^2; b) Cd; W_S = = 0.5 (I), 1 (II) and 2 (III) J/mm^2.

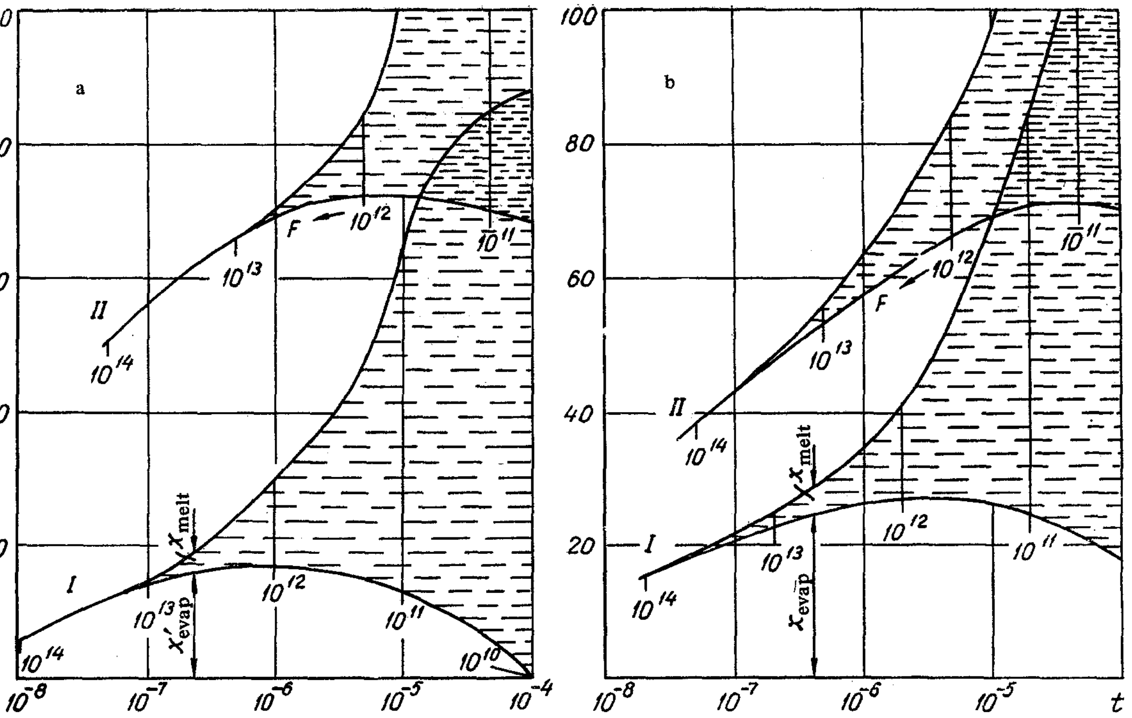

Fig. 2. Effect of the time (t, sec) a pulse of the energy W_s acts on a metal on the final position of the evaporation front and the melting isotherm (x'_{evap}, $x'_{evap} + x_{melt}$, μ; F, W/m^2): a) W; $W_s = 1$ (I) and 5 (II), J/mm^2; b) Cu; $W_s = 2$ (I) and 5 (II), J/mm^2.

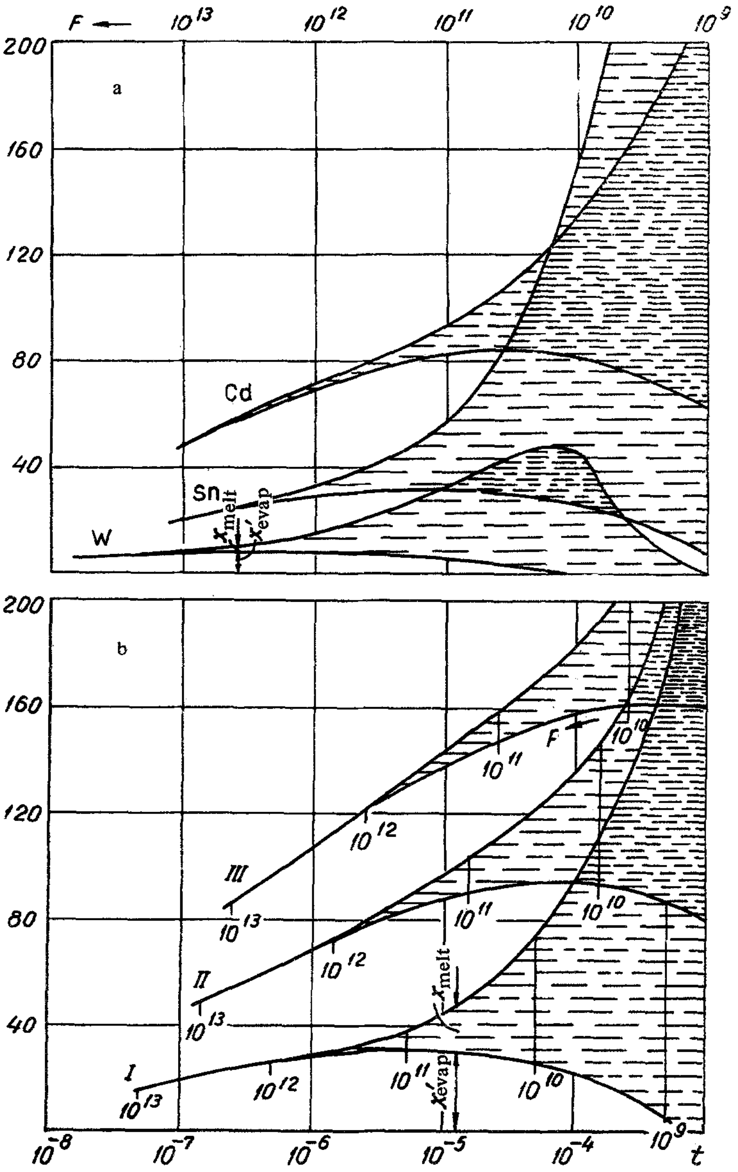

Fig. 3. Effect of the time (t, sec) a pulse of the energy W_s acts on a metal on the final position of the evaporation front and the melting isotherm (x'_{evap}, $x'_{evap} + x_{melt}$, μ; F, W/m^2): a) Cd, Sn, W; W_s = 1 J/mm^2; b) Pb; W_s = 0.5(I), 1.5 (II) and 2.5 (III), J/mm^2.

$$-\frac{\partial T(0,\ t)}{\partial x} = \frac{F}{ac_V} - \frac{F_0}{ac_V}\exp\left[-\frac{T_m}{T(0,\ t)}\right];$$

$$\frac{\partial T(\infty,\ t)}{\partial x} = 0,$$

$$T(x,\ 0) = T_0;\quad 0 \leqslant x \leqslant \infty;\quad 0 \leqslant t \leqslant t_p\,. \tag{4}$$

The effect of evaporation on the development of this process is that for given F it limits increase in the temperature of the front and ensures that the process will shift to the steady-state mode after a certain transition time has elapsed. In this mode, problem (3)–(4) becomes

$$aT''(x) + v^* T'(x) = 0, \tag{5}$$

$$-T'(0) = \frac{F}{ac_V} - \frac{r_V v^*}{ac_V},\quad F'(\infty) = 0, \tag{6}$$

$$T(0) = T^*,\quad T(\infty) = T_0, \tag{7}$$

where T* and v* are the steady-state values of the temperature and of the front velocity.

Solving (5), allowing for (7), and then integrating (5) over x within the limits of the semibounded region with use of (6) and (7), we obtain the following two equations:

$$T(x) = (T^* - T_0)\exp\left(-\frac{v^*}{a}x\right) + T_0, \tag{8}$$

$$v^*[r_V + c_V(T^* - T_0)] = F. \tag{9}$$

The first equation defines the temperature field in the steady-state mode, and the second equation expresses the energy conservation law in this mode. Eliminating T* from (9) with the aid of (2), we arrive at the following transcendental equation for the steady-state velocity:

$$\frac{F}{v^*} - \frac{c_V T_m}{\ln v_0 - \ln v^*} = r_V - c_V T_0. \tag{10}$$

System of equations (8)–(10) is completely solvable. In particular, we can calculate v* and T* from (9) and (10). The values of these quantities are shown in Table 1 for different flux densities and for one metal—cadmium.

From (8) we can obtain the depression in the melting isotherm $T(x_{\text{melt}}) = T_{\text{melt}}$ with respect to the front:

$$x_{\text{melt}} = \frac{a}{v^*}\ln\frac{T^* - T_0}{T_{\text{melt}} - T_0}. \tag{11}$$

As follows from the data in Table 1 and from Eq. (11), with increase in the flux density F the depression of the melting isotherm changes according to a law similar to $x_{\text{melt}} \sim F^{-1}$. Thus, for flux densities of 10^{11} and 10^{12} W^2/m^2, x_{melt} is respectively 8.4 and 1.35 μ for cadmium. A similar phenomenon is also observed for other metals.

Equations (5–11) describe the process in the steady-state mode. Before entering this mode, a transition process develops in the system; information as to its duration and the manner in which it develops can only be obtained by solving problem (3)–(4). Since this problem is essentially nonlinear its solution was obtained on an electronic computer. Some of the solution results, which also include the transition process, are shown in Figs. 1–3 where x_{melt} denotes the depression in the melting isotherm and a $x'_{\text{evap}} = \int_0^{t_p} v(t)\,dt$ — the evaporation-front displacement within the pulse time. These figures show the results due to the action of a pulse of fixed energy $W_s = Ft_p$ on a metal for different introduction conditions, i.e., for different flux densities and different corresponding pulse durations (W_s is expressed in J/mm^2).

As follows from the figures, the action of a pulse of the energy W_s for a short time interval is associated solely with the evaporation breakdown of the metal since $x'_{\text{evap}} \gg x_{\text{melt}}$ and $x_{\text{melt}} \approx 0$. With increase in the duration of action of the energy W_s the depression of the melting isotherm rises, conditions are created when $x_{\text{melt}} \approx x'_{\text{evap}}$, followed by conditions when $x_{\text{melt}} > x'_{\text{evap}}$, and lastly with further increase in pulse duration $x'_{\text{evap}} \to 0$, while x_{melt} remains large for some time and then drops to zero. Thus, with an increase in the duration of a pulse of fixed energy, the quantity $(x'_{\text{evap}} + x_{\text{melt}})$ reaches a maximum for some $t = t_m$, and then drops to zero (Figs. 2a and 3a). Thus, for Cu, $t_m = 5 \cdot 10^{-5}$ sec for $W_s = 0.5$ J/mm^2 and $t_m = 10^{-5}$ sec for W. It was also noted that as the pulse energy increases the maximum in $(x'_{\text{evap}} + x_{\text{melt}})$ increases and shifts toward larger times. The maximum in $(x'_{\text{evap}} + x_{\text{melt}})$ was not obtained by us for low-melting metals; however, it undoubtedly exists for $t \geqslant 10^{-3}$ sec.

The results of machine solution showed that the transition time essentially depends on the flux density. For $F = 10^{11}$, 10^{12}, and 10^{13} W/m^2, t_{trans} is respectively $2.5 \cdot 10^{-6}$, $4 \cdot 10^{-8}$, and $1.5 \cdot 10^{-9}$ sec for cadmium. Thus, for high-power pulses, the transition time is considerably shorter than the pulse duration, and the thermophysical process in this case can be considered according to the conditions of the steady-state mode. It also follows from the smallness of the transition time that for high-power pulses the fluctuation in flux density F and the associated fluctuation in the quantities v*, T*, and x_{melt} will actually take place in one phase.

Under high-power pulse action the melting isotherm in the steady-state mode closely adjoins the evaporation front and moves together with it into the metal at high speed. Such a phenomenon would also be observed with the melting front if the heat of fusion L_V were allowed for in the statement of the corresponding problem. In this case, for the steady-state mode instead of (9) we can obtain

$$v^*[L_V + r_V + c_V(T^* - T_0)] = F. \tag{12}$$

Since for high flux densities $c_V(T^* - T_0) \geqslant r_V$ and for all metals $L_V \ll r_V$, it is clear that no major difference occurs when the heat of fusion is allowed for during the action of a high-power pulse. For low flux densities, however, the transition process last for a time on the order of the pulse duration. For the low evaporation-front velocities here this causes the heat of fusion L_V to become one of the determining factors in the process. For a low-power pulse ($F < 10^9$ W/m^2) we must solve the corresponding problem, allowing for L_V, and this naturally leads to somewhat lower values of ($x'_{evap} + x_{melt}$) than predicted by the solution to problem (3)–(4) under such conditions.

We now consider the thermal effect of a pulse discharge on a metal. This is more complex than the action of a laser beam or plasma jet since the metal, upon discharge, is simultaneously subjected to surface and volume thermal effects. The former can be expressed in terms of the thermal-flux density

$$F = \xi j^*, \tag{13}$$

where j^* is the current density in the discharge channel at the electrode surface, and ξ is a coefficient with different expressions for the cathode and anode [7]. In a one-dimensional approximation, the power of the volume source can be expressed as

$$w = \rho(T) j^2(x) = w(T, x), \tag{14}$$

where ρ is the electrode resistivity. The corresponding one-dimensional problem can be formulated as follows:

$$\frac{\partial T(x, t)}{\partial t} = a\frac{\partial^2 T(x, t)}{\partial x^2} + v_0 \exp\left[-\frac{T_m}{T(0, t)}\right]\frac{\partial T(x, t)}{\partial x} + c_V^{-1} w(T, x); \tag{15}$$

$$-\frac{\partial T(0, t)}{\partial x} = \frac{F}{a c_V} - \frac{F_0}{a c_V}\exp\left[-\frac{T_m}{T(0, t)}\right]; \quad \frac{\partial T(\infty, t)}{\partial x} = 0$$

$$(T(x, 0) = T_0; \quad 0 \leqslant x \leqslant \infty; \quad 0 \leqslant t \leqslant t_p). \tag{16}$$

When the process becomes steady-state, the power distribution of the volume source also becomes steady-state; in this case, this quantity can be represented as

$$w = w^* f(\delta x), \tag{17}$$

where w^* is the power of the volume source at the evaporation front and $f(\delta x)$ is a rapidly attenuating function satisfying the conditions

$$f(0) = 1; \quad f(\delta x_m) \ll 1; \quad f(\infty) = 0. \tag{18}$$

Here x_m denotes the characteristic dimension of the process. As this quantity it is convenient to take the depth of a micropore appearing on the electrode as a result of a single action of the migrating channel. According to the data of our experiments with copper electrodes, this depth is ~10 μ. According to these data, it is possible to associate the source (17) with the experimental results by means of the parameter δ.

While the pulse acts $T^* \gg T_{melt}$ and, consequently, $\rho^* \gg \rho_{melt}$, where ρ^* is the resistivity at the evaporation front and ρ_{melt} is the resistivity of the liquid phase at the melting point. In performing the familiar linear extrapolation on the grounds that the metals retain their characteristic metal conductivity [10] on transition to the liquid state, we can express ρ^* in terms of ρ_{melt} and then represent w^* as

$$w^* = \rho_{melt}[1 + \alpha(T^* - T_{melt})] j^{*2}, \tag{19}$$

where α is the temperature coefficient of resistance. Data on this quantity for some metals were presented in [11].

When the process becomes steady-state, problem (15)–(16) becomes

$$aT''(x) + v^* T'(x) + c_V^{-1} w^* f(\delta x) = 0, \tag{20}$$

$$-T'(0) = \frac{F}{v^* c_V} - \frac{r_V v^*}{a c_V}; \quad T'(\infty) = 0, \tag{21}$$

$$T(0) = T^*; \quad T(\infty) = T_0. \tag{22}$$

The solution to problem (20)–(22) is given by a specific expression of the function $f(\delta x)$. We limit ourselves to an approximation of this function by setting $f(\delta x) = \exp(-\delta x)$. This is not a rough approximation since $\exp(-\delta x)$ is rapidly attenuating and satisfies all remaining properties of $f(\delta x)$ according to (18). In such a case, by solving (20) while allowing for (22), and by integrating (20) over x in the semibounded region while allowing for (21) and (22), we obtain the following two equations

$$T(x) = \left[T^* - T_0 + \frac{w^*}{c_V(a\delta^2 - v^*\delta)}\right]\exp\left(-\frac{v^*}{a}x\right) - \frac{w^*}{c_V(a\delta^2 - a\delta)}\exp(-\delta x) + T_0, \tag{23}$$

$$v^*[r_V + c_V(T^* - T_0)] = F + \frac{w^*}{\delta}. \tag{24}$$

The first of these equations defines the temperature field in the steady-state mode while the second expresses the energy-conservation law in this mode. These equations are solved simultaneously with (2), (13), and (19).

Table 2

Boundary Values for Current Densities for an Essentially Volume-Heat Source for Pb, Sn, and Cu

j_1^*, A/cm^2			j_2^*, A/cm^3		
Pb	Sn	Cu	Pb	Sn	Cu
$6 \cdot 10^5$	$3 \cdot 10^6$	$6 \cdot 10^6$	$5 \cdot 10^7$	$9 \cdot 10^7$	10^8

For very high current densities, conditions can arise when $w^*/\delta > F$ in Eq. (24); this is equivalent to

$$j^* < \frac{\xi\delta}{\rho_{\text{melt}}[1 + \alpha(T^* - T_{\text{melt}})]} = j_2^*. \qquad (25)$$

Since the evaporation-front velocity becomes very high under such conditions, $v^*/a \gg \delta$ and, according to (23), the depression of the melting isotherm or the characteristic dimensions of the process will be determined solely by the volume-source distribution in the region, and not by the transfer of heat liberated at the front and within the metal to the interior of the electrode by electron heat conduction. Condition (25) shows that the volume source predominates both at the evaporation front and within the volume.

Moreover, the opposite conditions can exist in Eq. (24) when $F > w^*/\delta$. These conditions appear as soon as $j^* < j_2^*$, since $F \sim j^*$ and $w^* \sim j^{*2}$. For $F < w^*/\delta$ the volume source loses its effectiveness at the evaporation front, but retains it within the volume. However, for $v^*/a < \delta$, and this is equivalent to

$$j^* < \frac{a\delta}{\xi}\left[r_V + c_V\left(\frac{T_m}{\ln v_0 - \ln a\delta} - T_0\right)\right] = j_1^*, \qquad (26)$$

depression of the melting isotherm or the characteristic dimensions of the process are now defined not by the volume-source distribution but by electron heat conduction. Since when $v^*/a < \delta$, $w^*/c_V(a\delta^2 - v^*\delta) < (T^* - T_0)$, the temperature field (23), governed jointly by the surface and volume effects almost coincides with the temperature field (8), governed primarily by the surface thermal effect. Condition (26) consequently defines the current-density region in which the volume source can be assumed to be completely ineffective.

In the pulse-discharge mode, three regions can therefore be distinguished. In the first one ($j^* < j_1^*$), the volume source is ineffective both at the front and within the volume; in the second ($j_1^* < j^* < j_2^*$), it is ineffective at the front but effective within the volume; lastly, in the third ($j^* > j_2^*$), it predominates both within the volume and at the evaporation front. If we use (24) to express the energy liberated at the electrodes by the migrating discharge channel while the pulse acts,

$$W_s = \sum_{i=1}^{i=n}\left(F_i t_i + \frac{w_i^*}{\delta_i} t_i\right), \qquad (27)$$

it is necessary to eliminate the terms $(w_i^*/\delta_i)\,t_i$ from (27) in the first region and the terms $F_i t_i$ in the third region; in the second region, none of the terms can be discarded because they are equivalent, particularly near j_2^*.

Estimates of the boundary values for j_1^* and j_2^*, obtained by simultaneously solving Eqs. (2), (13), (24), (25), and (26) are given for some anode metals in Table 2.

As follows from Table 2, the boundary of the third region is determined by the rather high value of the current density j_2^*. Smaller values of this quantity can only be obtained by reducing the estimates of ξ; however, there is no basis for doing so. Allowance for deviation in the strong fields from Ohm's law also does not lead to any decrease, because the associated increase in resistance is only a few percent for $j = 10^8$ A/cm^2 [4].

For a current density $j > j_2^*$ the power of the volume source, particularly at the evaporation front, becomes extremely high and even during the time a short pulse acts the energy $w^* t_p$ is liberated, which exceeds the atom-bond energy r_V; this must lead to rupture of the entire volume of the material, for which we have only $w(x)\,t_p \ge r_V$. Such conditions set the limits of applicability of Eqs. (23)–(24) and a basically new approach is necessary for analysis of the resulting phenomena. This different approach is also necessary for other reasons, because the relaxation time approaches the pulse duration and the electron gas degenerates owing to the extremely high temperatures.

In conclusion, we note that one-dimensional formulation of our problems does not deprive them of satisfactory approximation to the actual process, particularly for high pulse powers. This is because under these conditions the evaporation front moves into the metal at a very high rate, thereby reducing the possibility of radial heat-flux propagation and causing a thermophysical process to develop which is very close to being one-dimensional.

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